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room should be made for their spiritual training as well, even at the sacrifice of Sanscrit or dancing. It all rests with the parents and their judgment of what is the really indispensable.

I am far from minimising the responsibility of the clergy, but I am not here to speak of that. I am far from saying that their responsibility is discharged; but honestly I do not know how, under present conditions, they can discharge it. There are only twelve hours in the day. And we do not always find our efforts to do something for the children taken so seriously as we could wish.

"*Like as a father pitieth his children.*" There is great room for pity towards the little ones, who are so absolutely at the mercy of the older people, and have to grow up just what we make them. God help us to be good ourselves, for to be with good men and to learn their thoughts is a chief thing, without which no instruction will serve. But God help parents, too, to consider that *instruction* also is needed, and that their children have souls to be fed, as well as bodies to be nourished.

GREATEST COMMON MEASURE.

BY MRS. BOOLE.

SOME months ago a child of not quite ten said to me, "I don't understand Greatest Common Measure. I can do the sums fast enough, as a parrot can talk, and they come right, but I don't know why they come right; and I can't see why people want to do the thing at all." Now this very subject, Greatest Common Measure (known in text books as G.C.M.) is one of the great regrets—I might almost say sorrows—of the mathematical idealist. For him the rule illustrates, in a compact form, some of the subtlest and most lovely principles of pure mathematical induction. To sit and think over its sequence of successive analyses is to him a joy something like that which a musician experiences in thinking over a grand fugue; and it does seem a pity that this source of intellectual inspiration should be so utterly closed as it is to the large majority of those who "work sums" in G.C.M. But no explanation, however careful and elaborate, seems to convey to the average mind any real conception of what the rule really means and implies. It struck me that the last phrase of my young friend's lament: "I can't see why people want to do the thing at all,"—gives the clue to many disappointments in explaining arithmetic, for, however simple and logical our proof of the validity of a rule may be, it will always give the sense of *jugglery*, of something not quite real and honest, unless the imagination be first prepared by a concrete and, so to speak, picturesque presentation of some reason "why people want to do the thing at all." We talked over the matter together, till I succeeded in inventing a picture-presentation of the need for Greatest Common Measure, which I have used since with satisfactory results. It is probable that the same illustration has occurred to other teachers, but I am sure that very many have not thought of it. Moreover, the picturesque preparation of the imagination is always best done in vacation, with one or two children (not a class), in an atmosphere of leisure and absence of intellectual effort; I offer it, therefore, rather to the non-scientific mother than to the technically trained class-teacher.

I. Suppose you are asked to invent a cornice pattern which will begin at one end of the long wall of a room and finish off exactly at the other end. Let the side of a page of paper represent the length of the room. Now then, how long shall we make our pattern? We might make it *half* the length of the room, or one quarter, or one third, or any fraction of the room-length which has 1 for numerator; we must not make it *two*-thirds, or *two*-fifths, or *three*-sevenths the length of the room; if we do, the pattern will not exactly fit in at the end. A fraction of the length which has 1 for numerator is what is called in arithmetic a *measure* of the length. To fit in exactly, the length of the pattern must be a *measure* of the length of the room.

Suppose, next, that we are asked to make the pattern as long as possible. Then we would make it the whole length of the room. The length itself is called "the *greatest* measure" of the length. If instead of the length we had to decorate the breadth (represented by the breadth of the paper) then our pattern must be one half, one third, one something or other, of the breadth, and the largest possible pattern would be as long as the whole breadth, *i.e.*, the "greatest measure" of the breadth.

But now, suppose we want to decorate *both* the length and breadth, and we want our pattern to begin and end in each corner of the room. The length of our pattern must be "a *common* measure" of the length and breadth of our paper.

By the time this point has been reached the child will be growing familiar with the terminology which so many find bewildering and repulsive. The next aim of the teacher should be, just at this point, to awaken his imaginative perception to see the complication of process and the limitation of possible answers, which are due to having two wall-lengths instead of one to consider. Make him feel the sudden up-cropping of an apparently unsurmountable difficulty before you proceed to show him how the difficulty may be surmounted.

Mark three corners of the page, respectively, A, B and C, and let A B be the shorter length. Measure off from C towards B a length, C D, equal to A B. Show the child that

as the pattern has to repeat exactly in A B, it must also repeat exactly in C D; therefore D must be one of the points where it ends and begins again; therefore it must fit exactly into the remainder of the long side, *i.e.*, into D B. Therefore all we have to think of now is to invent a pattern which will fit exactly into A B and D B. Any pattern which will so fit will also fit into B C; and no pattern which does *not* answer for A B and B D can possibly be made to answer for B C. All the common measures of A B and B D are likewise common measures of A B and B C, so that we may discard C D out of our problem for the present and simply fix our attention on finding the common measures of A B and B D.

Now measure off from A towards B a line equal to B D. It is clear that any pattern which will fit into A B and B D will also fit into this newly cut off piece. Possibly the length B D can be cut off more than once from the length A B; if so mark it off as often as you can. Each of the marks so made will be a point where the pattern leaves off and begins again. Call the last of them (the nearest to B) E. Now, as D and E represent points where the pattern leaves off, the problem has dwindled down to that of finding the C.M.'s of E B and B D. Repeat this process over and over till you come at last to a remainder which is a "measure" of the last but one remainder; this last remainder, then, is a measure of A B and B C. Any pattern which fits exactly (whether once or more times) into the last remainder will repeat exactly both in A B and in B C. Make sure that the child understands how to get "Common Measures" of the two sides, before you trouble him about the question which is the "Greatest."

As to the shape of the paper, it will be well, at first, to make A B equal four-fifths of B C. The problem then comes to an end at the first step, for B D is a C.M. of both lengths. More complicated examples must follow. In the second instance A B may be seven-ninths of B C, E B will be G.C.M. A B equal eleven-thirty-sevenths of B C is a good later example; B D must then be taken three times as long as A B.

II. One of the great difficulties about G.C.M. in arithmetic is due to the sense of hocus-pocus caused to a child by being told to discard the quotient of each successive division and attend only to the remainders; for (he naturally thinks), "surely the quotient of a division sum is more important than the remainder." It will be well, therefore, to make this point clear. When we come to carry out the design, we shall deal with the whole impartially; it would be dishonest to undertake to decorate (or in any way actually deal with) two walls, and then only manipulate two little samples in a corner. But while we are only finding out how to deal with the whole, the honesty or dishonesty of substituting little scraps for the whole depends simply on whether we can be sure that the samples give a true clue as to the right method of dealing with the whole. Just so if an expert is hired to sanitate the water of a pool, it would be dishonest if he showed a spoonful of pure water and left the rest of the pool foul; but there is no moral or philosophical objection to his putting a spoonful under the microscope for the purpose of ascertaining how the whole should be dealt with. This is of course not in any intellectual sense a parallel to the process for finding G.C.M., but it is a philosophically sound illustration of one point which causes perplexity; it does truly meet and neutralize the special cause of the sense of confusion.

III. Advantage might be taken of the rule for G.C.M. to introduce the pupil to yet another notion, the non-realization of which by ordinary students causes much of their difficulty in grasping the principles of mathematical philosophy. All mathematics consist essentially in substituting easy problems for such as are in themselves either actually unsolvable, or practically so, by reason of great cumbersomeness. "How many beads in twice three beads?" The infant counts, the adult remembers the answer; neither of them can be said to go through any *mathematical* process. "How many years in twice three million years?" We cannot *directly* ascertain the answer, we shall not live long enough. "How many years in twice thirty years?" Some of us might conceivably live to count that, but it would be very inconvenient. In these two cases we substitute for the hard question an easier one—"What is twice three?" (the answer to which we ascertained in infancy by counting beads), and we arrange to make that easy question reveal the answer to the hard one.

The art of mathematics consists in the skilful adapting of such arrangements to successive difficulties. What we call numeration is one series of such arrangements; but there are endless other series to be dealt with at the various stages of study. By ten years old a child has become so familiar with actual numeration that he performs its processes automatically and without realizing what he is doing; and when he is suddenly introduced to a new way of substituting easy questions for complicated ones, he is struck with a helpless sense of bewilderment, and feels that he is somehow juggling with his numbers and values. It is well, therefore, to pull together in his mind the different forms of mathematical operation, by pointing out to him that in a multiplication sum he says and thinks and counts "twice three," when his question really is, perhaps, about two thousand times three million; and that (just as legitimately) he speaks of and measures the pattern lengths for two small residuums of wall-length, when his real question was about the whole wall-lengths, and that in each case the legitimacy of the substitution depends solely on whether he can prove beforehand that the answer of the easy question gives the true clue to the answer of the more complicated one. What is meant by understanding a sum is, in reality, being able to satisfy oneself of the validity of each successive substitution of an easier question, or series of questions, for the one we are unable to directly solve.

I would suggest that the picture-presentation of Greatest Common Measure should be spoken of to the child in three successive vacations. I have accordingly marked the topics to be illustrated, I, II and III.

The principle of Least Common Multiple will be readily made intelligible by suggesting the problem:—Suppose we have two patterns not equal, nor one a measure of the other, and we have to fit one into the cornice and the other into the dado. Find the shortest length of wall into which both will exactly fit.

In G.C.M. two walls are given; find the length of the largest pattern that will fit into both. In L.C.M. two patterns are given; find the length of the shortest wall into which both will fit.

A child who goes up into the G.C.M. class with those two images well fixed in his mind, will be prepared to get all the good possible from the teacher's explanation of the actual rule.